Control Chart for Attributes

BEM3104 Engineering Quality Control

Engineering Management Program
Faculty of Engineering and Industrial Technology, SSRU

Introduction

- Many quality characteristics cannot be conveniently represented numerically.
- In such cases, each item inspected is classified as either **conforming** or **nonconforming** to the specifications on that quality characteristic.
- Quality characteristics of this type are called **attributes**.
- Examples are nonfunctional semiconductor chips, warped connecting rods, etc,.

Types of Control Charts

Control Charts for Variables Data

$\bar{X}$ and R charts: for sample averages and ranges.

$\bar{X}$ and s charts: for sample means and standard deviations.

Md and R charts: for sample medians and ranges.

$\bar{X}$ charts: for individual measures; uses moving ranges.

Control Charts for Attributes Data

p charts: proportion of units nonconforming.

np charts: number of units nonconforming.

c charts: count of nonconformities.

u charts: count of nonconformities per unit.

Attribute

- The term Attribute refers to those quality characteristics that conform to specifications or do not conform to specifications.
- Attribute are used:
  1. Where measurements are not possible.
  2. Where measurements can be made but are not made because of time, cost, or need.

Attribute (cont.)

- A nonconformity is a departure of a quality characteristic from its intended level or state that occurs with a severity sufficient to cause an associated product or service not to meet a specification requirement.
- Defect is concerned with satisfying intended normal, or reasonably foreseeable, usage requirement.

Attribute (cont.)

- Defect is appropriate for use when evaluation is in terms of usage.
- Nonconformity is appropriate for conformance to specifications.
- The term *Nonconforming Unit* is used to describe a unit of product or service containing at least one nonconformity.

Attribute (cont.)

- Defective is analogous to defect and is appropriate for use when unit of product or service is evaluated in terms of usage rather than conformance to specifications.
- Limitations of variable control charts: These charts cannot be used for quality characteristics which are attributes.

Attribute (cont.)

Types of Attribute Charts:

1. Nonconforming Units (based on the Binomial distribution): *p* chart, *np* chart.
2. Nonconformities (based on the Poisson distribution): *c* chart, *u* chart.

Type of Attribute Charts

***p* charts**

- This chart shows the fraction of nonconforming or defective product produced by a manufacturing process.
- It is also called the control chart for fraction nonconforming.

***np* charts**

- This chart shows the number of nonconforming. Almost the same as the *p* chart.

***c* charts**

- This shows the number of defects or nonconformities produced by a manufacturing process.

***u* charts**

- This chart shows the nonconformities per unit produced by a manufacturing process.

The P Chart

- The *P* Chart is used for data that consist of the proportion of the number of occurrences of an event to the total number of occurrences.
- It is used in quality to report the fraction or percent nonconforming in a product, quality characteristic, or group of quality characteristics.

The *p* charts

- In this chart, we plot the percent of defectives (per batch, per day, per machine, etc.).
- However, the control limits in this chart are not based on the distribution of rate events but rather on the binomial distribution (of proportions).

The P Chart

- It can be used to control one quality characteristic, as is done with X bar and R chart,
- Or to control a group of quality characteristics of the same type or of the same part,
- Or to control the entire product.
- It can be established to measure the quality produced by a work center, by a department, by a shift, or by an entire plant.

The P Chart

- It is frequently used to report the performance of an operator, group of operators, or management as a means of evaluating their quality performance.
- The subgroup size of the *P* chart can be either variable or constant.

Formula

- Fraction nonconforming:

  $p = (np)/n$

- where p = proportion or fraction nc in the sample or subgroup, n = number in the sample or subgroup, np = number nc in the sample or subgroup.

Example

- During the first shift, 450 inspection are made of book-of the month shipments and 5 nc units are found. Production during the shift was 15,000 units. What is the fraction nc?

$$p = (np)/n = 5/450 = 0.011$$

- The p, is usually small, say 0.10 or less.
- If $p > 0.10$, indicate that the organization is in serious difficulty.

p-Chart construction for constant subgroup size

- Select the quality characteristics.
- Determine the subgroup size and method
- Collect the data.
- Calculate the trial central line and control limits.
- Establish the revised central line and control limits.
- Achieve the objective.

Select the quality characteristics

The quality characteristic?

- A single quality characteristic
- A group of quality characteristics
- A part
- An entire product, or
- A number of products.

Determine the subgroup size and method

- The size of subgroup is a function of the proportion nonconforming.
- If $p = 0.001$, and $n = 1000$, then the average number nc, $np = 1$. *Not good, since a large number of values would be zero.*
- If $p = 0.15$, and $n = 50$, then $np = 7.5$, would make a good chart.

- Therefore, the selection subgroup size requires some preliminary observations to obtain a rough idea of the proportion nonconforming.

Collect the data

- The quality technician will need to collect sufficient data for at least 25 subgroups.
- The data can be plotted as a run chart.
- Since the run chart does not have limits, its is not a control chart.

Calculate the trial central line and control limits

- The formula:

$$UCL = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$

$$LCL = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$

- $\overline{p} = \frac{\sum np}{\sum n}$ = average of *p* for many subgroups

- *n* = number inspected in a subgroup

The p-chart for Constant Subgroup Sizes

- The Centerline and Control Limits

$$\text{Centerline}(p) = \bar{p} = \left[\frac{\text{Total number of defectives in all subgroups under investigation}}{\text{Total number of units examined in all subgroups under investigation}} \right]$$

$$\text{UCL}(p) = p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\text{LCL}(p) = p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

- **Construction of a p-chart**
  - An Example
    - As an illustration, consider the case of an importer of decorative ceramic tiles. Some tiles are cracked or broken before or during transit, rendering them useless scrap. The fraction of cracked or broken tiles is naturally of concern to the firm. Each day a sample of 100 tiles is drawn from the total of all tiles received from each tile vendor.

Daily Cracked Tiles

Day	*Sample Size*	*Number Cracked or Broken*	*Fraction*
1	100	14	0.14
2	100	2	0.02
3	100	11	0.11
4	100	4	0.04
5	100	9	0.09
6	100	7	0.07
7	100	4	0.04
8	100	6	0.06
9	100	3	0.03
10	100	2	0.02
11	100	3	0.03
12	100	8	0.08
13	100	4	0.04
14	100	15	0.15
15	100	5	0.05
16	100	3	0.03
17	100	8	0.08
18	100	4	0.04
19	100	2	0.02
20	100	5	0.05
21	100	5	0.05
22	100	7	0.07
23	100	9	0.09
24	100	1	0.01
25	100	3	0.03
26	100	12	0.12
27	100	9	0.09
28	100	3	0.03
29	100	6	0.06
30	100	9	0.09
Totals	3,000	183	

Centerline (p) = 183/3000 = 0.061

$$UCL(p) = 0.061 + 3\sqrt{\frac{0.061(1-0.061)}{100}} = 0.133$$

$$LCL(p) = 0.061 - 3\sqrt{\frac{0.061(1-0.061)}{100}} = -0.011 = 0.000$$

- For a stable process, the probability that any subgroup fraction will be outside the three-sigma limits is small.
- Also, if the process is stable, the probability is small that the data will demonstrate any other indications of the presence of special causes of variation.
- But if the process is not in a state of statistical control, the control chart provides an economical basis upon which to search for and identify indications of this lack of control.

- For this p-chart -- or, in fact, for any of the attribute control charts -- the exact probabilities that a stable process will generate points indicating a lack of control are impossible to calculate because even a stable process exhibits variation in its mean, dispersion, and shape.
- Nevertheless, the exact value of these probabilities is not too important for ordinary applications; what is important is the fact that they are small.
- Therefore, if a point does lie beyond the upper or lower control limits, we will infer that it indicates a lack of control.
- Additionally, for p-charts, the six other rules for out-of-control points described in Chapter 6 can all be applied. In order to do so, we need to compute the boundaries for the A, B, and C zones.

7.4.2 Construction of a p-chart

– Boundary between upper zones B and C

$$= \bar{p} + \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$$

In our example this value is 0.061 + 0.024 = 0.085

– Boundary between lower zones B and C

$$= \bar{p} - \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$$

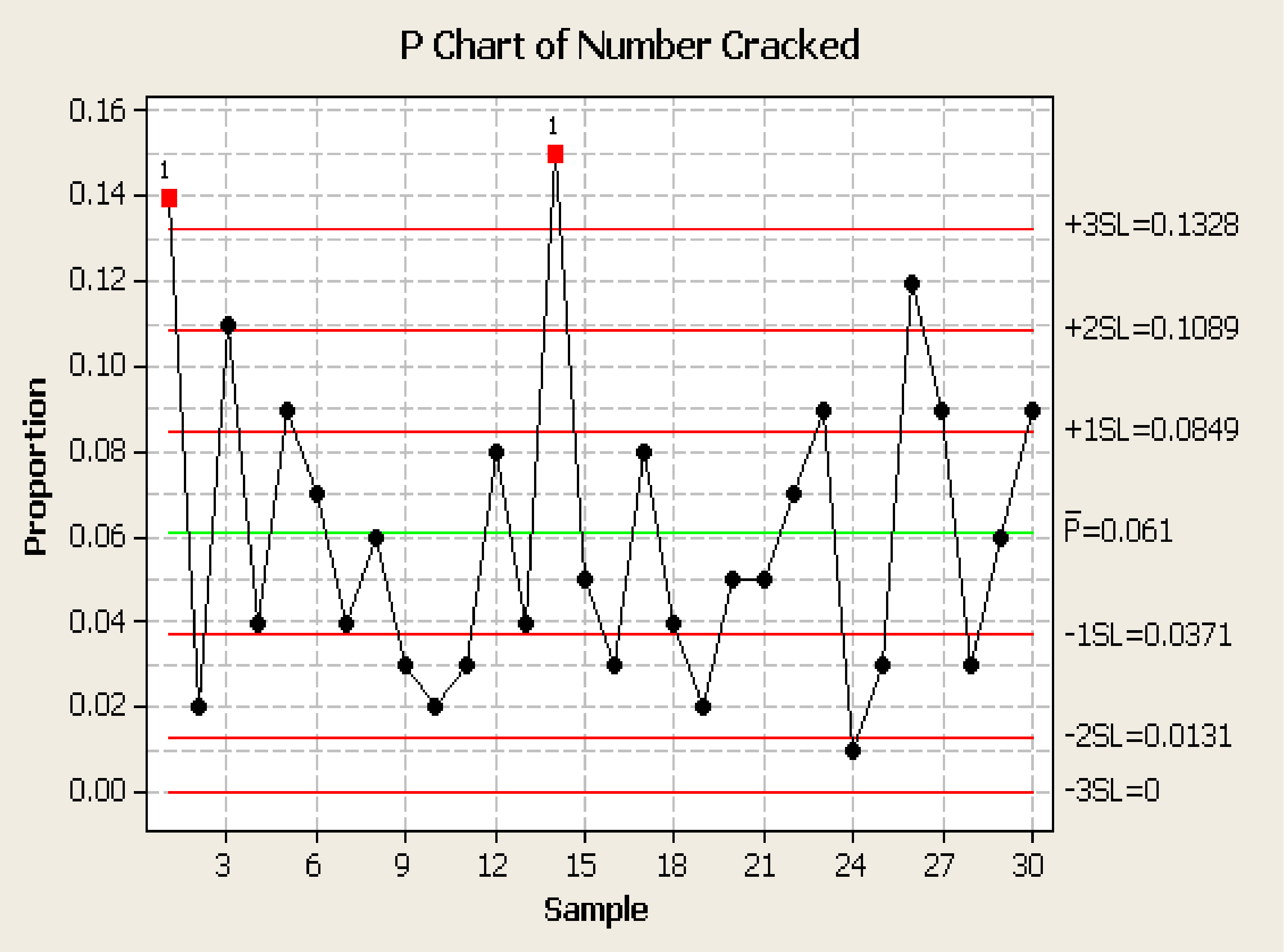
In our example this value is 0.061 - 0.024 = 0.037.

– Boundary between upper zones A and B

$$\bar{p} + 2\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.061 + 2(0.024) = 0.109$$

– Boundary between lower zones A and B

$$\bar{p} - 2\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.061 - 2(0.024) = 0.013$$



P Chart of Number Cracked
Proportion
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
1
1
+3SL=0.1328
+2SL=0.1089
+1SL=0.0849
P̄=0.061
-1SL=0.0371
-2SL=0.0131
-3SL=0
3
6
9
12
15
18
21
24
27
30
Sample

np Chart

- The *np* chart is almost the same as the *p* chart.

Central line = np_o

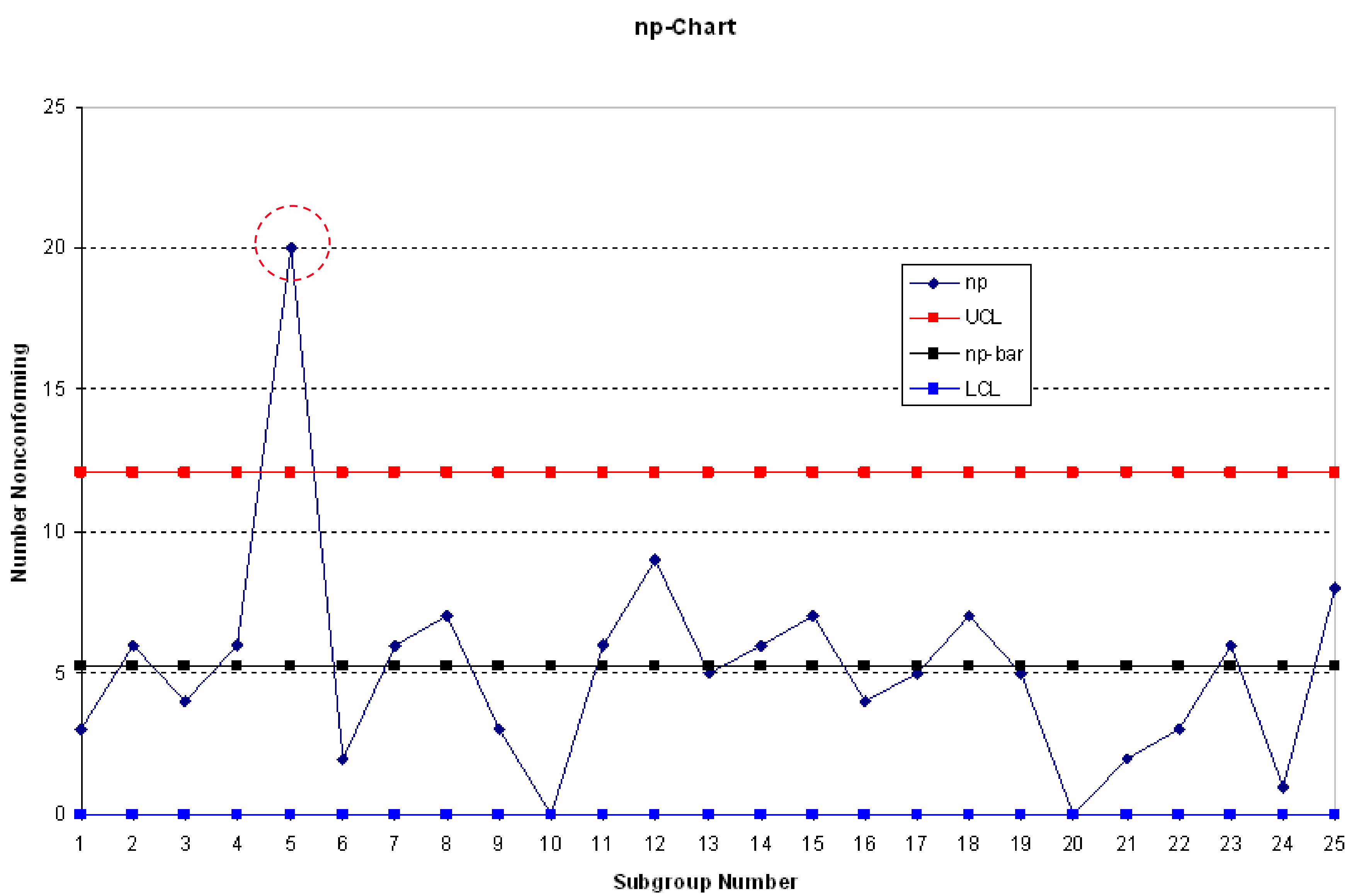
$$UCL = np_o + 3\sqrt{np_o(1 - p_o)}$$

$$LCL = np_o - 3\sqrt{np_o(1 - p_o)}$$

- If p_o is unknown, it must be determined by collecting data, calculating UCL, LCL.

Example

Subgroup	*n*	*np*	UCL	*np*-bar	LCL
1	300	3	12.0	5.24	0.0
2	300	6	12.0	5.24	0.0
3	300	4	12.0	5.24	0.0
4	300	6	12.0	5.24	0.0
5	300	20	12.0	5.24	0.0
21	300	2	12.0	5.24	0.0
22	300	3	12.0	5.24	0.0
23	300	6	12.0	5.24	0.0
24	300	1	12.0	5.24	0.0
25	300	8	12.0	5.24	0.0


np-Chart
Number Nonconforming
Subgroup Number
0
5
10
15
20
25
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
np
UCL
np-bar
LCL

c Chart

- The procedures for c chart are the same a s those for the *p* chart.
- If count of nonconformities, co, is unknown, it must be found by collecting data, calculating UCL & LCL.

$$UCL = \bar{c} + 3\sqrt{\bar{c}}$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}}$$

$\bar{c} = \frac{\sqrt{c}}{g}$ = average count of nonconformities

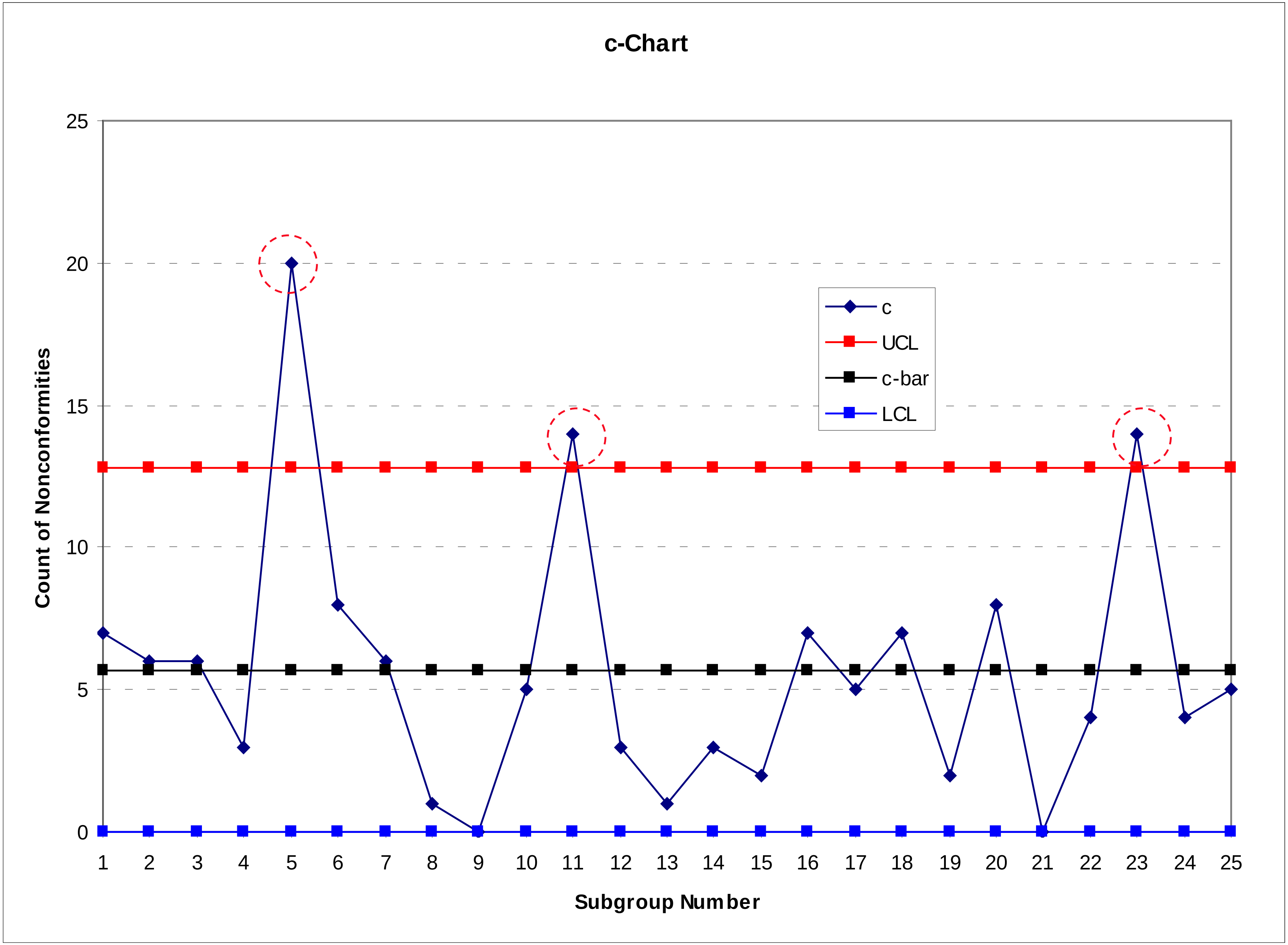
Example

$$\bar{c} = \frac{\sqrt{c}}{g} = \frac{141}{25} = 5.64$$

$$UCL = 5.64 + 3\sqrt{5.64} = 12.76$$

ID Number	Subgroup	*c*	UCL	*c*-bar	LCL
MY102	1	7	12.76	5.64	0
MY113	2	6	12.76	5.64	0
MY121	3	6	12.76	5.64	0
MY125	4	3	12.76	5.64	0
MY132	5	20	12.76	5.64	0
MY143	6	8	12.76	5.64	0
MY150	7	6	12.76	5.64	0
MY152	8	1	12.76	5.64	0
MY164	9	0	12.76	5.64	0
MY166	10	5	12.76	5.64	0
MY172	11	14	12.76	5.64	0
MY267	22	4	12.76	5.64	0
MY278	23	14	12.76	5.64	0
MY281	24	4	12.76	5.64	0
MY288	25	5	12.76	5.64	0

$$LCL = 5.64 - 3\sqrt{5.64}$$
$$= -1.48 = 0$$


c-Chart
Count of Nonconformities
Subgroup Number
0
5
10
15
20
25
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
c
UCL
c-bar
LCL

Revised

- Out-of-control: sample no. 5, 11, 23.

$$\bar{c}_{new} = \frac{\sqrt{c} - c_d}{g - g_d} = \frac{141 - 20 - 14 - 14}{25 - 3} = 4.23$$

$$UCL = c_o + 3\sqrt{c_o} = 4.23 + 3\sqrt{4.23} = 10.40$$

$$LCL = c_o - 3\sqrt{c_o} = 4.23 - 3\sqrt{4.23} = -1.94 = 0$$

u Chart

- The u chart is mathematically equivalent to the c chart.

$$u = \frac{c}{n}$$

$$\overline{u} = \frac{\sum c}{\sum n}$$

$$UCL = \overline{u} + 3\sqrt{\frac{\overline{u}}{n}}$$

$$LCL = \overline{u} - 3\sqrt{\frac{\overline{u}}{n}}$$

Example

$$\bar{u} = \frac{\sum c}{\sum n} = \frac{3389}{2823} = 1.20$$

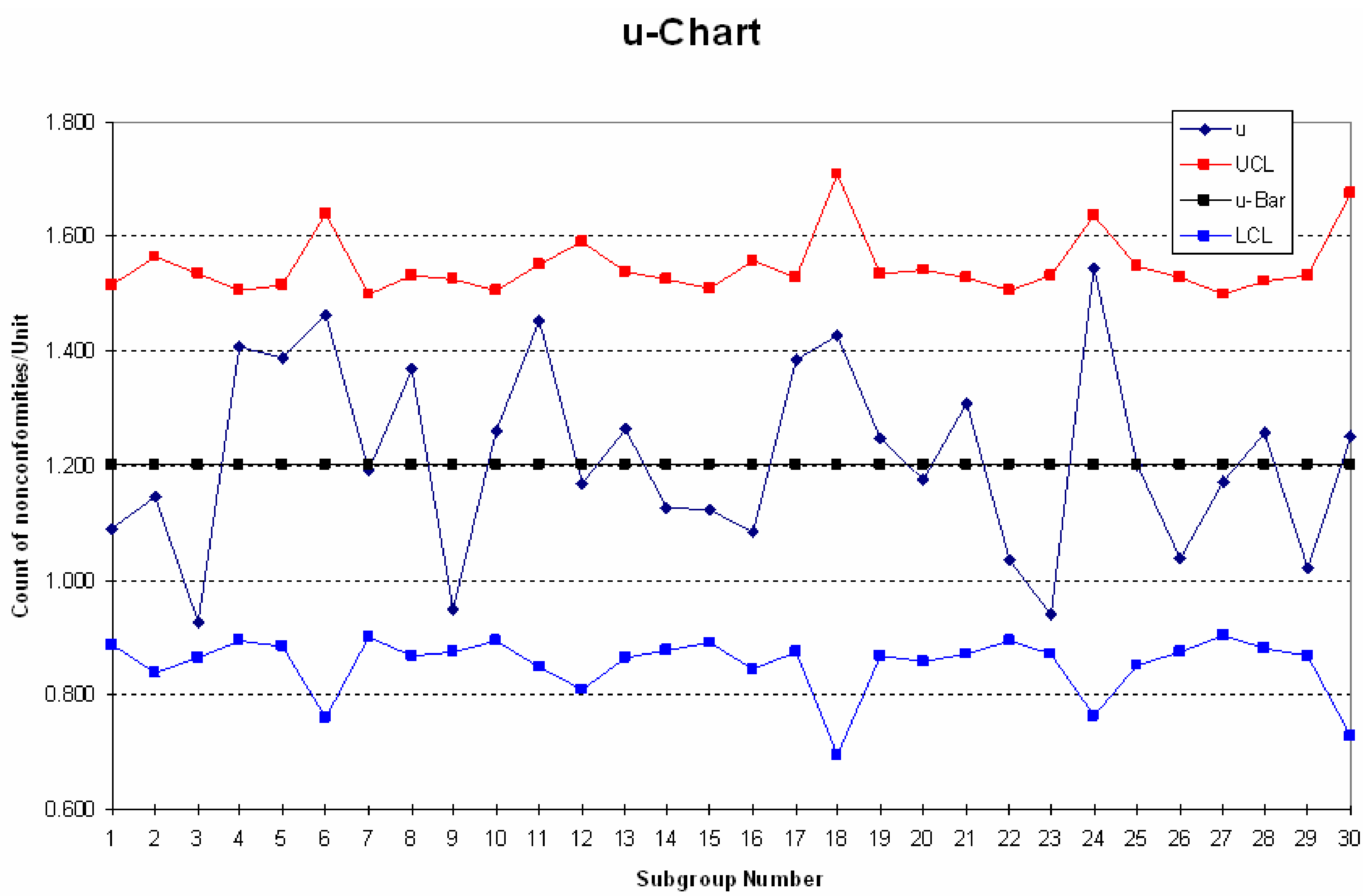
ID Number	Subgroup	*n*	*c*	*u*	UCL	*u*-Bar	LCL
30-Jan	1	110	120	1.091	1.51	1.20	0.89
31-Jan	2	82	94	1.146	1.56	1.20	0.84
1-Feb	3	96	89	0.927	1.54	1.20	0.87
2-Feb	4	115	162	1.409	1.51	1.20	0.89
3-Feb	5	108	150	1.389	1.52	1.20	0.88
4-Feb	6	56	82	1.464	1.64	1.20	0.76
28-Feb	26	101	105	1.040	1.53	1.20	0.87
1-Mar	27	122	143	1.172	1.50	1.20	0.90
2-Mar	28	105	132	1.257	1.52	1.20	0.88
3-Mar	29	98	100	1.020	1.53	1.20	0.87
4-Mar	30	48	60	1.250	1.67	1.20	0.73

- For January 30:

$$u_{Jan\,30} = \frac{c}{n} = \frac{120}{110} = 1.09$$

$$UCL_{Jan\,30} = 1.20 + 3\sqrt{\frac{1.20}{110}} = 1.51$$

$$LCL_{Jan\,30} = 1.20 - 3\sqrt{\frac{1.20}{110}} = 0.89$$


u-Chart
1.800
1.600
1.400
1.200
1.000
0.800
0.600
Count of nonconformities/Unit
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
Subgroup Number
u
UCL
u-Bar
LCL